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SOLUTION BY W. E. HEAL, WHEELING, INDIANA.

Let the indefinite integral

$$\int \! dx \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} = \varphi(x)$$
, then $\int_0^x dx \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} = \varphi(x) - \varphi(0) = \frac{ny}{x}$

Differentiating the above eq'n, (1), remembering that $\varphi(0)$ is constant,

$$dx\sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]} = \frac{n(xdy-ydx)}{x^2}.$$
 (2)

Put $dy \div dx = p$, and (2) becomes $x^2 \checkmark [(1+p^2)] = npx-ny$. (3) Differentiating (3) we have

$$\frac{xpdp}{\sqrt{(1+p^2)}} + 2dx\sqrt{(1+p^2)} = ndp. \tag{4}$$

Divide by $2(1+p^2)^{\frac{1}{4}}$; $dx(1+p^2)^{\frac{1}{4}} + xpdp \div (1+p^2)^{\frac{1}{4}} = ndp \div 2(1+p^2)^{\frac{1}{4}}$. (5) Integrating (5) and determining the constant C = 0, we have

$$x(1+p^2)^{1/4} = \frac{n}{2} \int \frac{dp}{(1+p^2)^{1/4}}$$
, (6). Let $p = \frac{2^{1/4}v(1-\frac{1}{2}v^2)^{1/2}}{1-v^2}$, then (6) becomes

$$\begin{split} \frac{x}{(1-v^2)^{\frac{1}{2}}} &= \frac{n}{2^{\frac{1}{2}}} \int \frac{dv}{\sqrt{\left[(1-v^2)^3(1-\frac{1}{2}v^2)\right]}} = 2^{\frac{1}{2}} n \left[\frac{v(1-\frac{1}{2}v^2)^{\frac{1}{2}}}{(1-v^3)^{\frac{1}{2}}} \right. \\ &+ \frac{1}{2} \int \frac{dv}{(1-v^2)^{\frac{1}{2}}(1-\frac{1}{2}v^2)^{\frac{1}{2}}} - \int \frac{(1-\frac{1}{2}v^2)^{\frac{1}{2}}dv}{(1-v^2)^{\frac{1}{2}}} \right] = 2^{\frac{1}{2}} n \left[\frac{v(1-\frac{1}{2}v^2)^{\frac{1}{2}}}{(1-v^2)^{\frac{1}{2}}} + \frac{1}{2}F(\frac{1}{2}2^{\frac{1}{2}},v) \right] \end{split}$$

 $-E(\frac{1}{2}2^{\frac{1}{2}}, v)$] (7), where F and E are elliptic funct's of the 1st and 2nd ord's.

But
$$v = \frac{[(1+p^2)^{\frac{1}{2}}-1]^{\frac{1}{2}}}{(1+p^2)^{\frac{1}{2}}}$$
, and from (3) $p = \frac{n^2y + x[n^2(x^2+y^2)-x^4]}{x(n^2-x^2)}$.

Substituting these values in (7) we have the equation of the curve.

[The solution of 306, by Prof. Johnson, of 307, by Prof. Seitz, and of 308, by Mr. Adcock, will be published in No. 4.]

PROBLEMS.

- 309. By Prof. Beman.—In a given circle, find the vertices of the inscribed square, pentagon, octagon, and decagon by using the dividers alone.
- 310. By Prof. Edmunds.—Required the locus of vertices of a right angled spherical triangle whose legs pass through two fixed points given on the surface of the sphere.
- 311. By Prof. Casey.—A uniform circular plate is placed with its centre upon a prop, to find at what points on its circumference three given weights p, q, r must be attached that it may remain at rest in a horizontal position.
- 312. By Prof. Scheffer.— To find the area of the loop of the curve $y^3+x^2y-axy+bx^2=0$.